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# LARGE-SCALE MOTION HYPOTHESIS FOR A GAS—FLUID FLOW

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UDC 532.529.5

The experimental information concerning the local characteristics of gas—fluid flow accumulated in recent years provides a stimulus for the development of semiempirical methods for studying such flows.

In the work available until now, a two-phase mixture is often considered as a locally homogeneous fluid, to which the assumptions concerning the hydrodynamics of single-phase flows are applied [1, 2]. At the same time, large-scale fluctuations in the hydrodynamic quantities, velocity, pressure, and gas content, are no longer considered.

The role of large-scale fluctuations in the gas—fluid flow is demonstrated in [3] via an analysis of the balance of turbulent energy, according to which there is a transformation of energy in the fluctuating (macroscopically fluctuating) motion into the energy of averaged motion in a two-phase flow. The presence of large-scale fluctuations must be taken into account in the starting equations for the conservation of mass, momentum, and energy in two-phase flows.

Many Soviet and foreign researchers have been concerned with the construction of a system of differential equations that describes the motion of multiphase systems. An analysis of the best-known work is given in the reviews [4, 5].

One of the basic questions is the choice of scales for averaging the integral conservation equations. Most researchers consider the two-phase mixture as an incompressible fluid with dispersed solid particles. It is assumed beforehand that both components are present in the volume of the mixture that is being averaged and, in addition, the volume concentration does not depend on the size of the averaging volume down to infinitely small volumes.

As a result, correlations that contain concentration fluctuations appear in the averaged equations.

This approach can be used for mixtures in which the dimensions of the occlusions are significantly less than the scales over which the spatial average is performed.

In the motion of gas—fluid mixtures in the plug regime, the characteristic size of the occlusions is comparable with the scale of the flow (pipe diameter). For such a flow, it may be assumed that at any time the volume over which the averaging is performed is occupied by one of the phases.

In this case, correlations that contain concentration fluctuations vanish and averaging the equations of the conservation of mass and momentum reduces to the equations in [6]:

$$\frac{\partial}{\partial t}(\rho_i\alpha_i) - \nabla(\rho_i\alpha_i\mathbf{U}_i) = 0,$$

$$\frac{\partial}{\partial t}(\rho_i\alpha_i\mathbf{U}_i) = -(\nabla\rho_i\mathbf{U}_i)\,\alpha_i\mathbf{U}_i + \rho_i\alpha_i\mathbf{g} + \nabla(\alpha_i\mathbf{P}_i) + \nabla(\alpha_i\mathbf{T}_i).$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 114-120, July-August, 1980. Original article submitted September 25, 1979.

  

The tensor $P_i$ in the cartesian coordinate system (m; n = x; y; z) has the form

$$P_{imn} = -p_i\delta_{mn} + \mu_i(\partial U_{im}/\partial n + \partial U_{in}/\partial m)$$

($\delta_{mn}$ is the unit tensor).

The components of the tensor of the second rank

$$T_{imn} = -\rho_i\overline{u'_{im}u'_{in}} \tag{1}$$

cannot be viewed as arising only from turbulent fluctuations, since deviations from the average values can also have a nonrandom nature.

In order to calculate these components, it is necessary to introduce particular assumptions concerning the relationship between the fluctuating velocity components and the field of the time-averaged hydrodynamic flow quantities.

The energy spectra of the fluctuations in pressure [3, 7] and friction on the wall [8] indicate the presence of two types of fluctuations in the hydrodynamic quantities in a two-phase pipe flow: small scale, arising from the overall instability similar to the fluctuations in a single-phase flow, and large-scale fluctuations

$$f = \overline{f} + f'_t + f'_l, \tag{2}$$

where f is the effective instantaneous value of the hydrodynamic quantity; $f'_t$ is the small-scale (purely turbulent) component of the fluctuations; $f'_l$ is the large-scale component.

For a wide range of variations in the flow rate parameters, the maximum in the spectral functions of the fluctuations [3] lies in the region of low frequencies, which indicates that most of the energy goes into large-scale fluctuations.

Substituting (2) into (1), we obtain

$$T_{imn} = -\rho_i\left(\overline{u'_{imt}u'_{int}} + \overline{u'_{imt}u'_{in_l}} + \overline{u'_{im_l}u'_{int}} + \overline{u'_{im_l}u'_{in_l}}\right). \tag{3}$$

Since small-scale fluctuations and large-scale fluctuations have different characters, it is reasonable to assume that large-scale motion, caused by fluctuations in concentration [3], interacts weakly with single-phase turbulence. According to this assumption, the second and third terms on the right side of Eq. (3) equal zero and the equation takes the form

$$T_{imn} = -\rho_i\left(\overline{u'_{imt}u'_{int}} + \overline{u'_{im_l}u'_{in_l}}\right). \tag{4}$$

The first term on the right side of (4) has the same meaning as for the usual single-phase turbulence: the shearingstress, caused by turbulent velocity fluctuations. In addition, it follows from (4) that in a gas–fluid flow the shearing stresses increase due to the additional large-scale mixing.

The expression for shearing stress in a two-phase flow with a longitudinal velocity component u and transverse component v neglecting viscosity can be written in the form

$$\tau = -\alpha_1\rho_1\left(\overline{u'_{1t}v'_{1t}} + \overline{u'_{1_l}v'_{1_l}}\right) - \alpha_2\rho_2\left(\overline{u'_{2t}v'_{2t}} + \overline{u'_{2_l}v'_{2_l}}\right), \tag{5}$$

where $\alpha_{1(2)}$ is the probability for the appearance of the phase 1 (2) at the given point (local gas content). The subscript 1 relates everywhere to the liquid phase and the subscript 2 to the gas phase.

The correlation moments, containing the small-scale velocity fluctuations, can be expressed with the help of known relationships from the semiempirical theory of turbulence in a homogeneous liquid, for example, with the help of Prandtl's mixing length hypothesis:

$$-\overline{u'_{it}v'_{it}} = \varkappa_i^2 y^2\left(\frac{d\bar{u}_i}{dy}\right)^2,$$

where i = 1,2; $\varkappa_t$ is von Karman's constant; y is the distance from the wall.

Assuming that the velocity profiles of both phases and the velocity profile of the mixture $\bar{u}_c = \alpha_1\bar{u}_1 + \alpha_2\bar{u}_2$ are similar, we obtain

$$-\overline{u'_{it}v'_{it}} = \varkappa_t^2 y^2 \left(\frac{\bar{u}_i}{\bar{u}_c}\right)^2 \left(\frac{d\bar{u}_c}{dy}\right)^2.$$

We will also use the mixing length theory for estimating the correlation moments of the form $\overline{u'_{il}v'_{il}}$. One of the first efforts along these lines was [9], wherein additional shearing stresses in the liquid phase, arising from the motion of a gas bubble relative to the surrounding liquid, are examined. The time-averaged product of large-scale velocity fluctuations and distance from the wall have the form [9]

$$\overline{(-u'_{il}v'_{il})_y} = |\overline{u'_{il}}|\,|\overline{v'_{il}}|\,[1-\alpha_i(y_1)], \tag{6}$$

where $y_1$ is the distance from the wall at which there is a gas bubble, giving rise to the additional (large-scale) mixing.

The relations for the additional shearing stresses in the liquid phase are obtained in [9] with an analysis of the motion of a spherically shaped bubble in an infinite volume of liquid. The magnitude of these stresses is determined in terms of the values of the diameter of the bubble and its velocity relative to the surrounding liquid averaged over the cross section of the channel.

In the plug regime for the flow of a mixture, large gaseous occlusions move in a restricted space, bounded by the walls of the pipe. The theory developed in [9] does not take this into account. In addition, the model in [9] is inconvenient due to the fact that the dependence of the diameter of the bubble on the flow rate and physical characteristics of the flow are unknown at present. The experimental study of this dependence presents great difficulties. For this reason, in order to obtain computational formulas, it is more convenient to express the shearing stresses in terms of the known characteristics of the two-phase flow.

Let us examine the interphase surface moving with a constant velocity c without changing its shape. The equations of the interphase surface $a(x, t)$ ($a$ is the distance from the center of the channel) can be represented in the form of a function of a single variable $\xi = x - ct$. The function $a = a(\xi)$ is periodic; its period equals the length of the gas–liquid plug $l_{gf}$.

The gas particles move relative to the interphase surface with the velocity $\bar{u}_2 - c$, while the particles of liquid move with the velocity $\bar{u}_1 - c$. Due to such relative motion, additional mixing arises. In the case that $\bar{u}_i - c = 0$, there is no additional mixing within the phase i and the large-scale velocity fluctuations in this phase are absent.

During plug flow of a gas–liquid mixture, almost the entire gas phase is concentrated in large occlusions (vapor locks). Following the model proposed in [9], we will assume that the flow of liquid around the vapor lock occurs as a flow of an ideal liquid around a cylindrical body with a variable thickness $a(\xi)$.

In this case, the streamlines in the liquid diverge at some distance $Y(\xi)$ from their initial position. If the distance at which the liquid retains its initial momentum is proportional to Y, the mixing length in the large-scale fluctuation motion $l_l(\xi) \sim Y(\xi)$. For a two-phase flow, in which the vapor locks are symmetrical relative to the pipe axis (the plug flow in vertical pipes, as well as in pipes with arbitrary orientation with Froude number, greater than the self-similar value), the characteristic dimensions are the distance from the wall y and the pipe diameter D, as well as $a(\xi)$. Since the shape of the interface surface is unknown, we assume that

$$l_l(\xi) \sim Y(\xi) = y\frac{a(\xi)}{R}, \tag{7}$$

where R = D/2 is the radius of the pipe. The expression (7) satisfies the boundary conditions

$$l_l(\xi) = 0 \quad \text{for} \quad y = 0 \text{ and } a(\xi) = 0, \qquad (8)$$
$$l_l(\xi) = a(\xi) \quad \text{for} \quad y = R.$$

Averaging (7) over the period $a(\xi)$, we obtain

$$l_l = y\sqrt{\varphi_2}, \qquad (9)$$

where for a round pipe with radius R

$$\varphi_2 = \frac{\bar{a}^2}{R^2}, \quad a = \frac{1}{l_l}\int_{l_l} a(\xi)\,d\xi.$$

Following the mixing length theory, we write

$$|u'_{1l}| \sim |v'_{1l}| \sim l_l \frac{d(\bar{u}_1 - c)}{dy}. \qquad (10)$$

The experimental data [10] indicate that the local concentration of phases with plug flow of a gas–liquid mixture shows almost no change across the cross section of the pipe. The layer near the wall, where $\alpha_2$ increases sharply from 0 to $\langle\alpha_2\rangle = \varphi_2$, is an exception.

Setting $\alpha_i = \varphi_i$, expression (6) with the help of (9) and (10) and the hypothesis concerning the similarity of the velocity profiles of both phases introduced earlier can be transformed into the form

$$-\overline{u'_{1l}v'_{1l}} = \varkappa_l^2 y^2 \varphi_2^2 \left(\frac{\bar{u}_1 - c}{\bar{u}_c}\right)^2 \left(\frac{d\bar{u}_c}{dy}\right)^2, \qquad (11)$$

where $\varkappa_l$ is a constant.

A similar expression can be written for large-scale velocity fluctuations in the gas-phase, which occur with the motion of liquid occlusions:

$$-\overline{u'_{2l}v'_{2l}} = \varkappa_l^2 y^2 \varphi_1^2 \left(\frac{\bar{u}_2 - c}{\bar{u}_c}\right)^2 \left(\frac{d\bar{u}_c}{dy}\right)^2. \qquad (12)$$

Substituting (6), (11), and (12) into (5), we obtain

$$\tau = \rho_c A y^2 \left(\frac{d\bar{u}_c}{dy}\right)^2, \qquad (13)$$

where $A = \left\{\frac{\varphi_1\rho_1}{\rho_c}\left[\varkappa_t^2\left(\frac{\bar{u}_1}{\bar{u}_c}\right)^2 + \varkappa_l^2\left(\frac{\bar{u}_1 - c}{\bar{u}_c}\right)^2\right] + \frac{\varphi_2\rho_2}{\rho_c}\left[\varkappa_t^2\left(\frac{\bar{u}_2}{\bar{u}_c}\right)^2 + \varkappa_l^2\left(\frac{\bar{u}_2 - c}{\bar{u}_c}\right)^2\right]\right\}$; $\quad \rho_c = \varphi_1\rho_1 + \varphi_2\rho_2.$

During plug flow of a mixture, there always occurs at the walls of a pipe a liquid layer with thickness $\delta_\lambda$, within which the flow is laminar. Setting $\alpha_2 = 0$ for $0 \leqslant y \leqslant \delta_\lambda$, $\alpha_2 = \varphi_2$ for $y > \delta_\lambda$, and defining the dynamic speed and dynamic length

$$u_* = \sqrt{\tau/\rho_c}, \; l_* = \nu_1/u_*, \qquad (14)$$

we resume

$$\delta_\lambda = \gamma l_*, \; \bar{u}_{1\lambda} = \beta_1\bar{u}_{c\lambda}/\varphi_1 = \gamma u_*, \qquad (15)$$

where $\bar{u}_{1\lambda}$ is the speed of the liquid at the external boundary of the laminar sublayer and $\gamma$ is a numerical factor. Furthermore, just as for the single-phase flow, $\gamma = 11.5$ [11].

Integrating (13) taking into account (14) and (15), we obtain the velocity distribution in the maximum across the cross section of the pipe

$$\frac{\bar{u}_c}{u_*}=\frac{1}{\sqrt{A}}\ln\frac{yu_*}{\nu_1}+\left(\frac{\varphi_1}{\beta_1}\gamma-\frac{1}{\sqrt{A}}\ln\gamma\right). \qquad (16)$$

In order to determine the hydraulic resistance coefficient of the mixture we used the expression [3]

$$\tau=\lambda_c\rho_c B\,\langle\bar{u}_c\rangle^2, \qquad (17)$$

where

$$B=\frac{1}{\rho_c}\left(\frac{\beta_1^2}{\varphi_1}\rho_1+\frac{\beta_2^2}{\varphi_2}\rho_2\right).$$

With the help of (14), (16), (17), it is possible to obtain in the usual way [11]

$$\lambda_c=\left\{\frac{0.813\sqrt{B}}{\sqrt{A}}\left[\lg\left(\mathrm{Re}\sqrt{\lambda_c}\right)+\lg\sqrt{B}-2.47\right]+4.07\frac{\varphi_1}{\beta_1}\sqrt{B}\right\}^{-2}, \qquad (18)$$

where $\mathrm{Re}=\langle\bar{u}_c\rangle D/\nu_1$.

In the case of the flow of a mixture in a rough pipe, we will write the conditions at the top of the protrusions in the form

$$y=k_e,\quad \bar{u}_1=\bar{u}_{1k}=\frac{\beta_1}{\varphi_1}\bar{u}_{ck}=\Phi\left(\frac{k_e u_*}{\nu_1}\right).$$

Integrating (13), we obtain

$$\frac{\bar{u}_c}{u_*}=\frac{1}{\sqrt{A}}\ln\frac{y}{k_e}+\frac{\varphi_1}{\beta_1}\Phi\left(\frac{k_e u_*}{\nu_1}\right).$$

The hydraulic resistance coefficient for a rough pipe is given by

$$\lambda_c=\left[\frac{0.813\sqrt{B}}{\sqrt{A}}\left(\lg\frac{D}{2k_e}-0.65\right)+\frac{3\varphi_1}{\beta_1}\sqrt{B}\right]^{-2}. \qquad (19)$$

For $\beta_2 = 0$, formulas (18) and (19) become the formulas for the hydraulic resistance coefficients for single-phase flow [11].

The absence of a sufficient quantity of experimental data concerning the profiles of the averaged velocities of mixtures in a plug flow does not permit determining the constant $\varkappa_l$ from (16). For this reason, $\varkappa_l$ was computed by comparing (18) and (19) with experimental data [3]. For constant $\varkappa_l$, we obtain the numerical value $\varkappa_l = 1.73$.

For practical calculations of plug flow for a two-phase mixture, we obtain the interpolation formula

$$\lambda_c=\left[(3-1.26a)-2a\lg\left(\frac{2k}{D}+\frac{18.7}{\mathrm{Re}\sqrt{\lambda_c}}\right)\right]^{-2}, \qquad (20)$$

where

$$a=\left[1+18.8(1-K)^2\frac{\beta_2^2}{\beta_1^2}\right]^{-1/2};\quad K=\varphi_2/\beta_2.$$

Analyzing relation (16), we note that the distribution of averaged velocities in a two-phase flow does not satisfy a universal logarithmic law. The difference consists of the fact that the coefficient in the term containing the logarithm and the free term in Eq. (16) are not constants, but depend on the ratio of the volume rates and the true volume concentrations of the components of the mixture.

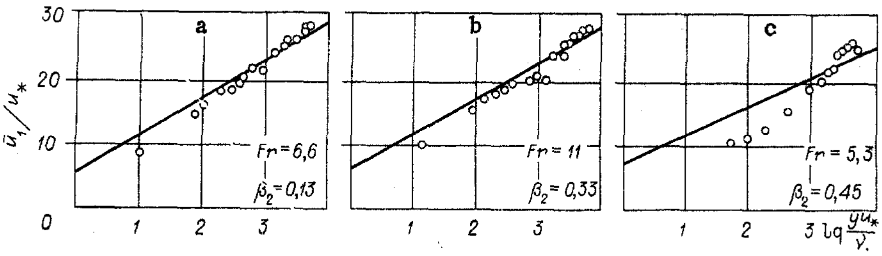


Fig. 1

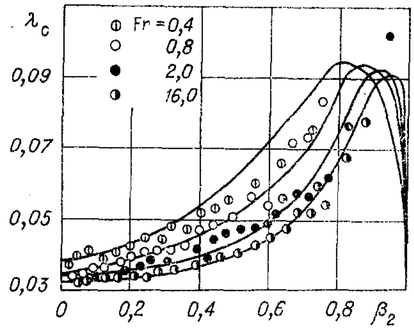


Fig. 2

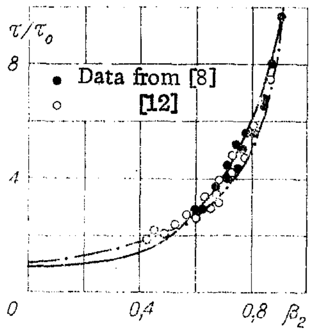


Fig. 3

The conditions for which the profile of the averaged velocities of the mixture are more uniform than for turbulent flow of a homogeneous liquid are determined by the inequality $A > \varkappa_t^2$. For $\rho_1 \gg \rho_2$, this corresponds to

$$\beta_2 > \frac{0.32}{3(1-K)+0.16(1+K)}.$$

For an air–water mixture with p = 0.1 MPa (K = 0.81) we obtain $\beta_2 > 0.372$. We obtain the velocity distribution of the liquid phase in a two-phase plug flow by substituting into (16) the obvious equality $(\beta_1/\varphi_1)\bar{u}_c$.

For $\rho_1 \gg \rho_2$

$$\frac{\bar{u}_1}{u_*} = \frac{1}{\sqrt{0.16+3(1-K)^2\beta_2^2/\beta_1^2}} \ln\frac{yu_*}{\nu_1} + \left(\gamma - \frac{1}{\sqrt{0.16+3(1-K)^2\beta_2^2/\beta_1^2}} \ln\gamma\right). \tag{21}$$

We note that the profile $u_1(y)$ is more uniform than the velocity profile in a single-phase turbulent flow, since the radicand in the first part of Eq. (21) exceeds $\varkappa_t^2 = 0.16$ for all $\beta_2 \neq 0$.

For the same reason, the hydraulic resistance coefficient $\lambda_c$, computed according to formula (20), is always greater than the hydraulic resistance coefficient for the flow of a homogeneous liquid [11] and, furthermore, this difference increases with increasing $\beta_2$.

Comparison of the results of the calculation of the velocity profiles of the fluid according to (21) with the data of V. P. Odnoral, obtained at the Institute of Technical Physics of the Siberian Division of the Academy of Sciences of the USSR with the help of the electrodiffusion method, shows good agreement between experiment and calculation (Fig. 1).

Figure 2 shows a comparison of the computational results obtained according to the proposed technique (solid line) with the experimental data [3], obtained for different values of Froude's number. As can be seen, the calculation agrees best with the experiment in the region of the developed flow of the mixture for Fr > 4.

Comparison of (20) with the presently known empirical methods for computing plug flow is complicated by the fact that most techniques do not make use of the concept of the coefficient of hydraulic resistance of the mixture. In those cases when this concept is introduced, the computational equation for determining the pressure loss is written differently in different methods.

Figure 3 shows a comparison of the proposed method (solid line) with the experimental data [8, 12] in the coordinates $\tau/\tau_0$, $\beta_2$. This figure also shows the computational results obtained with the method in [13] (dot–dash line). It is evident that the computational results obtained agree satisfactorily with the indicated experimental data in the entire region of the existence of plug flow.

The authors are grateful to their coworkers in the Laboratory of Physical Hydrodynamics at the Institute of Technical Physics of the Siberian Division of the Academy of Sciences of the USSR for information concerning the measurements of velocity profiles.

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